Nonlinear Resonant Interactions in Internal Cavity Flows

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FOLLOWING the suggestion of Flandro and Jacobs[1] that periodic vortex shedding from internal obstructions could produce significant oscillating pressure levels in turbulent duct flow, Culick and Magiawala,[2] Dunlop and Brown,[3] and Brown et al.[4] presented a variety of results verifying the phenomenon of acoustic energy production by vortex shedding and interaction. In further investigation of some of the flow mechanisms involved in acoustic energy production by vortex shedding, Isaacson and Marshall[5] presented measurements of oscillating velocity frequency spectra which indicated that in certain flow regimes, velocity oscillations occur which satisfy the frequency resonant interaction. This Note presents results of bispectral cross-power measurements of trios of frequencies occurring in an internal cavity in a turbulent flow which provide additional evidence that these trios of frequencies are of quadratic interaction origin.

These measurements were made in a subsonic wind tunnel which is driven by a centrifugal fan, driving air into a settling chamber which is 0.61 m on each side and 1.22 m long. The settling chamber is segmented and contains four damping screens, which act to reduce the turbulence of the flow into the test section. A converging section connects the settling chamber to the test section which is 0.178 m on each side and 2.44 m long.

The internal flow cavity is made up of four plexiglas restrictors, with the forward set of restrictors located 1.89 m downstream of the entrance to the test section. A schematic diagram of the flow cavity is shown in Fig. 1. For the measurements reported here, the gap between the forward pair of restrictors was 53.3 mm and the distance between the two pairs of restrictors was 101.6 mm.

A TSI Model 1050 constant-temperature hot-film anemometer, together with a TSI Model 1055 linearizer, and a TSI Model 1057 signal conditioner were used for these measurements. The single sensor, located a distance of 16 mm behind the forward restrictor pair along the flow axis of zero vertical velocity, was used to measure the velocity frequency spectrum presented in Fig. 2. The output of this sensor was also used to generate the bispectral measurements indicated in Fig. 3.

The frequency spectra were obtained with a Hewlett-Packard Model 3590A wave analyzer and an HP Model 3594A sweeping local oscillator. The analog output of the analyzer was digitally processed with a Hewlett-Packard Model 2240A measurement and control processor, with an HP-85 computer as the controller. The data were plotted on a Hewlett-Packard Model 7225A plotter.

Note that the cavity configuration used in this experiment produces two converging, almost parallel, shear layers at the forward baffles. Kelly[6] has shown that shear layers of this type produce instabilities of both subharmonic and superharmonic frequencies relative to the most unstable frequency determined by the Orr-Sommerfeld equations. Phillips[7] has shown that waves of this type can be turned away from the critical region of the shear layer and propagated outward as $f = N\cos$ where N is the stability frequency and θ the inclination to the horizontal. We thus have in this experiment a configuration analogous to the situation with the intersection of two surface water waves. Phillips[8] and Bretherton[9] have shown that the interaction of two wave trains with primary frequencies ω_1 and ω_2 can give rise to interaction internal waves of frequencies $\omega_1+\omega_2$ and $\omega_1-\omega_2$. Different wave components can thus undergo resonant interactions and produce wave components with wave numbers and frequencies formed by the sums and differences of those of the primary components.

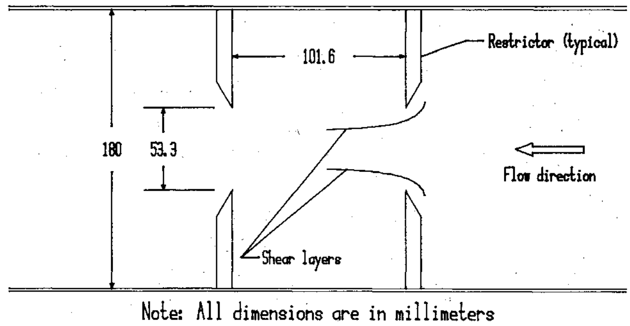

Fig. 1 Schematic diagram of flow cavity configuration.

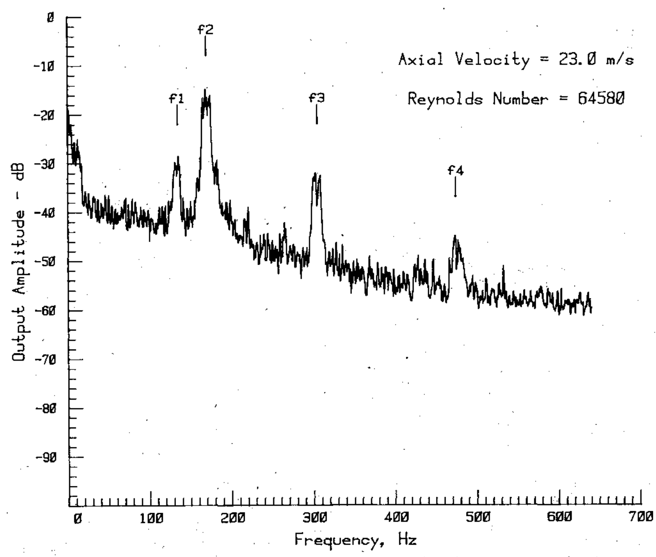

Fig. 2 Axial velocity frequency spectrum for a sensor located 16 mm behind the forward restrictor pair on the center streamline (Reynolds number is based on axial mean velocity and gap distance).

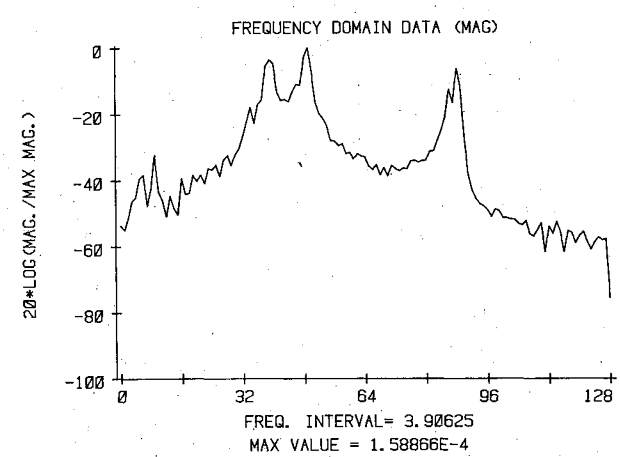

Fig. 3 Computer-generated bispectrum for frequencies f_1, f_2, and f_3 indicated in Fig. 2.

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The frequencies shown in Fig. 1 satisfy the first selection rule for nonlinear interactions,

$$f_1 \pm f_2 \pm f_3 = 0 \text{ and } f_2 \pm f_3 \pm f_4 = 0$$

If the fundamental frequency of the unstable shear layer is denoted as f_2, then note that the frequencies satisfy the rule

$$f_1 = f_3 - f_2 \text{ and } f_4 = f_3 + f_2$$

At this point, it is not clear whether the waves at frequencies f_1 an f_4 are nonlinearly produced from the waves at frequencies f_3 and f_2 or whether they are spontaneously excited independent waves associated with the acoustic characteristics of the wind tunnel. In order to distinguish between nonlinear coupled waves and linear independent waves, Kim et al.[10,11] have shown that digital implementation of a higher order spectrum known as the bispectrum is very useful in identifying nonlinear waves from a self-excited fluctuation spectrum.

Kim and Powers[10] have shown that digital implementation of the bispectrum provides a method for distinguishing between spontaneously excited and coupled modes by measuring the degree of phase coherence between modes. The bispectrum $B(\omega_1,\omega_2)$ corresponds to a two-dimensional Fourier transform of a second-order correlation function $C(\tau_1,\tau_2) = \langle x(t)x(t+\tau_1)x(t+\tau_2)\rangle$, where the bracket indicates the expectation value. Although the bispectrum is equivalent to a two-dimensional Fourier transform of $C(\tau_1,\tau_2)$ it may also be written as

$$B(\omega_1,\omega_2) = \lim_{T\to\infty} (1/T)\langle x(\omega_1)x(\omega_2)x^*(\omega_1+\omega_2)\rangle$$

where $x(\omega)$ is the Fourier transform of $x(t)$, T the time duration of the $x(t)$ signal, and the asterisk represents a complex conjugate.

The bispectrum will be zero unless waves are present at the frequencies ω_1, ω_2, and $\omega_1+\omega_2$ and, simultaneously, a phase coherence must be present between these waves. If the sum and difference frequency waves are generated through a nonlinear interaction, then a phase coherence exists. The statistical averaging will not lead to a zero value of the bispectrum. To carry out the demonstration for the first trio of frequencies shown in Fig. 2, measurements of the bispectral correlation coefficient were carried out. The bispectral correlation coefficient is defined as

$$\left\langle \frac{u'(f_1,t)}{\langle u'^2(f_1)\rangle^{1/2}} \cdot \frac{u'(f_2,t')}{\langle u'^2(f_2)\rangle^{1/2}} \cdot \frac{u'(f_3,t'')}{\langle u'^2(f_3)\rangle^{1/2}} \right\rangle$$

where $u'(t)$ is the velocity oscillation associated with each frequency and the angled brackets indicate the expectation value. To evaluate this parameter for the first three frequencies shown in Fig. 2, the Hewlett-Packard Model 2594A sweeping local oscillator was sequentially set at each of the first three frequencies. The HP Model 2240A measurement and control processor was used to take 256 digital data points, at the rate of 1000 samples/s of each of the wave forms from the band pass filters of the analyzer. Each resulting set of time-domain data was then fast-Fourier transformed into the frequency domain by the HP-85. These three sets of data were then used to compute the bispectral correlation coefficient. The resulting time-domain data were then transformed into the frequency domain which is presented in Fig. 3. The waveform analysis procedure developed by Hewlett-Packard Corporation for the HP-85 was used in these calculations.

The results presented in Fig. 3 indicate a high degree of correlation between the three frequencies, which is a requirement for quadratic interactions.

In addition to the frequency selection rule, the nonlinear terms of the Navier-Stokes equation introduce the wave number selection rule as[8,9]

$$k_1 \pm k_2 \pm k_3 = 0$$

where the wave number is defined as

$$k = 2\pi f / c_r(f)$$

where $c_r(f)$ is an appropriate wave phase velocity at that frequency. In order to show conclusively that these waves are of nonlinear interaction origin, it will be necessary to show that the wave number selection rule is also satisfied.

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A Fuzzy Algorithm to Compute Transonic Profile Flow

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Introduction

WHEN the governing equation of a system is nonlinear and no analytical solution is obtained, one has to depend on some iteration scheme. It is a general observation that the convergence of these schemes depends on the prechoice of the starting values. The convergence is ensured and the scheme will be faster if the starting solution more or less approximates the true solution. This approximation is,

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